

Example 1

$\pi_1 = L^a V^b \rho^c F$	$\pi_2 = L^a V^b \rho^c \mu$
$(L)^a (L^1 T^{-1})^b (M^1 L^{-3})^c (M^1 L^1 T^{-2}) = M^0 L^0 T^0$	$(L)^a (L^1 T^{-1})^b (M^1 L^{-3})^c (M^{-1} L^{-1} T^{-1}) = M^0 L^0 T^0$
Solving, $a=-2, b=-2, c=-1$	Solving, $a=-1, b=-1, c=-1$
$\pi_1 = L^{-2} V^{-2} \rho^{-1} F = \frac{F}{\rho^1 U^2 L^2} = C_F$	$\pi_2 = L^{-1} V^{-1} \rho^{-1} \mu = \frac{\mu}{\rho^1 U^1 L^1} = Re^{-1}$

$$\frac{F}{\rho^1 U^2 L^2} = f\left(\frac{\rho U L}{\mu}\right)$$

Now to look at that this is called a  $\pi$ —theorem it is a very easy concept that though please do not have a conclusion with the  $\pi$  means 3.14 it is a non dimensional form of p is okay. So if it is that I can have a 3 variables okay that is the length velocity and the  $\pi$  and the force these are called the repeating variables length velocity and viscosity I will put the repeating variables and with the F.

Similar way the length velocity  $\rho$  will be the repeating variability and then I substituting the  $\mu$  okay I do not know what power x point of length, velocity and the  $\rho$  will make it a non-dimensional form of  $\pi_1$  or I do not know what is the power experiment could be there to make a non-dimensional form of length, velocity density and the dynamic viscosity to make a  $\pi_2$  will be non-dimensional.

$$\pi_1 = L^a V^b \rho^c F$$

$$(L)^a (L^1 T^{-1})^b (M^1 L^{-3})^c (M^1 L^1 T^{-2}) = M^0 L^0 T^0$$

Solving,  $a=-2, b=-2, c=-1$

$$\pi_1 = L^{-2} V^{-2} \rho^{-1} F = \frac{F}{\rho^1 U^2 L^2} = C_F$$

Then I will take  $\pi_2$  to this which is a reciprocal of Reynolds number okay that is equal to the Reynolds number. That means one of the  $\pi_1$  is the functions of a  $\pi_2$  that is what you have written it only we are just the reciprocal of Reynolds numbers you make it here another Reynolds number format. So this is what our non-dimensional form this is what is our non-dimensional form.

$$\pi_2 = L^a V^b \rho^c \mu$$

$$(L)^a (L^1 T^{-1})^b (M^1 L^{-3})^c (M^{-1} L^{-1} T^{-1}) = M^0 L^0 T^0$$

Solving,  $a=-1, b=-1, c=-1$

$$\pi_2 = L^{-1} V^{-1} \rho^{-1} \mu = \frac{\mu}{\rho^1 U^1 L^1} = Re^{-1}$$

$$\frac{F}{\rho^1 U^2 L^2} = f\left(\frac{\rho U L}{\mu}\right)$$

(Refer Slide Time: 26:03)

Dimensionless groups in fluid dynamics

The following variables may be essential for the complete description of most of the fluid flow phenomena.

- Pressure
- Length
- Viscosity
- Surface tension
- Speed of sound
- Acceleration due to gravity
- Density
- Velocity

$p$   
 $L$   
 $\mu$   
 $\sigma$   
 $\alpha$   
 $g$   
 $\rho$   
 $V$

Three basic dimensions are required to describe above eight variables, so  $(8-3) = 5$  independent dimensionless groups can be formed with these variables.

So similar way let go for what are the variable in general in fluid flow we will get it. As I told it earlier that when you talk about fluid flow problems the mostly we talk about the velocity and the pressure field okay that is the basic things velocity and the pressure field. Then when you talk about that you know it there would be gravity force which will really taking care by acceleration due to gravity.

The length dimensions will be there then comes out to be mass functions will be the density will be within the mass the viscosity will define the flow resistance. So 2 is what is very important is called the surface tensions when you have an interface of 2 liquids 2 fluids you will have a surface tension. Same way as we discussed many of the times the speed of sound also matters for us to know it flow numbers or incompressibility of the flow or compressibility of the slow.

So all to tell if we look it any fluid flow problems okay can have all the dimensions or some of this that means pressures, length, viscosity, surface tension speed of the sound, acceleration due to gravity then Density and the velocity these are the prime variables okay. See if you look at how many are there?8 variables 3 basic variables. So that all these 8 variables can we make a 5 independent dimensional groups that is the concept of the fluid mechanics okay.

So any most of the fluid flow problems okay. Now you do in a very extensive different way but most of the fluid flow problems we can define with these 8 variables that means we can group them into a 5 independent dimensional groups so what I did.

(Refer Slide Time: 28:14)

Dimensionless groups in fluid dynamics

Some of the dimensionless groups formed from previous mentioned variables of fluid phenomena are:

- Reynolds number

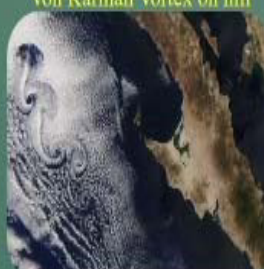
$$R_r = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho VL}{\mu}$$

- Froude number


$$F_r = \frac{\text{Inertia force}}{\text{Gravity force}} = \frac{V}{\sqrt{Lg}}$$


(GATE 2013 CE)

Von Karman Vortex on hill

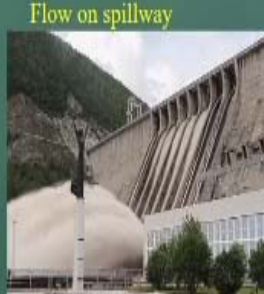


Smoke cloud from mouth







Flow on spillway



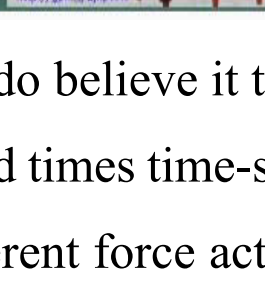
Cloud wake



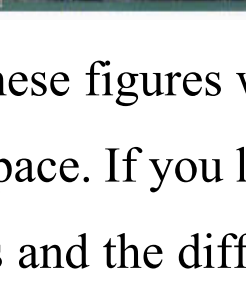
Boat wake



Wave cloud pattern in the wake of the ice (underneath in the southern Indian ocean)



Kelvin wake pattern generated by a small boat



Now i can see the show you figures I do believe it these figures will talk many things what I may not express with these so-called limited times time-space. If you look at the things what it happens that when we have fluid flows in different force acts and the different force dominate at different conditions and when you do the fluid flow problems, we should point out which one is dominated.

And you would try to solve for the dominated case so that is the way we look in the fluid flow problems to solve to find out or to judge which is the forces are dominating it like for examples when you have a laminar turbulent proof like this case okay you could see the laminar turbulent what i discussed in the first class okay by showing the what is exciting and all. So these are what?



These are 2 forces are dominated here one is inertia forces the momentum blocks and another easy viscous component.

These 2 forces are dominant and those 2 forces the ratio is Reynolds number. You can compute this

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho V L}{\mu}$$

I am just leaving you to just do that exercise otherwise we will discuss when your assignment mode okay you can compute it that the inertia force by viscous force is a Reynolds numbers. That is what we can do it using virtual fluid concept also. So let us have that its a ratio between a fluid flow problems where inertia forces versus viscous force that one the  $\frac{\rho V L}{\mu}$  is defined as the Reynolds numbers.

So often you use the Reynolds numbers because though mostly we consider the inertia force by viscous force dominancy else you can see the vortex on the hills okay that is what we discuss a lot how the vortex formation happens in and very smoke just smoke cloud from mouth or you can have the formation of vortex settings all it depends upon what Reynolds numbers of the flow we have.

In a similar way if you go for the next numbers it is called Froude number it depends on the fluid flow where your gravity force also have a dominance as compared to the viscous force. Like for examples it will talk about this spill way here the viscous force are not that dominant the gravity force works dominancy levels. So we can have a ratio between inertia force by gravity force that is what is flow Froude numbers that is what we define it that is what is defined by this as non-dimensional.


So if you look it this is the velocity length into g of square root also the unit dimensions of the velocity. So it becomes non-dimensional so how to get its this component I think in this basic lectures I cannot complete that wave celerity concept here but I will tell it in higher classes if you go for postgraduates and all or the open channel flow definitely you know it how this  $\sqrt{Lg}$  comes a gravity force components.


The same process its happens wave cloud pattern also it happens n wave pattern generated by a small fluid you can look at this because its very interesting figures you can see this all sort of the flow dominancy. The top lines are the flow dominancy of inertia force that is what it drive the fluid versus the viscous force in this case the inertia force and the gravity force. I just repeat you please remember these 2 numbers because this is a very common numbers are called Reynolds numbers and Froude problem number.

$$Fr = \frac{\text{Inertia force}}{\text{Gravity force}} = \frac{V}{\sqrt{Lg}}$$


(Refer Slide Time: 32:29)

Dimensionless groups in fluid dynamics

- Weber number
 
$$W = \frac{\text{Inertia force}}{\text{Surface tension}} = \frac{\rho V^2 L}{\sigma}$$


Bubble forming
- Euler (cavitation) number
 
$$E_u = \frac{\text{Pressure force}}{\text{Inertia force}} = \frac{-\Delta p}{\rho V^2}$$


Cavitation forming in turbine



While bullet traveling in water

Now if you look at other 2 numbers we not many times we use it but nowadays the level of the fluid mechanics problems what we are solving it we look at these formulas. One is called where the surface tension dominancy is there. For examples you can look at this bubble formation all of this because of the survey tensions. So in this case if I proposition on this bubble formation then I should look it the Weber number is represents as a ratio between inertia force by surface tension.

That is the component what we are getting but you know there is a cavitation surface when they present at certain levels then what are the fluid forms which we discussed in the BC classes what we taught it in a lecture 1 lecture 2. Okay so what did happen in that case? We try to look at how

much pressure drops which generate us the cavitation process. Pressure drops the change in the pressure if the more the change in the pressure there is a likelihood to have a cavitation force divide by this inertia forces is defined by the Euler numbers or cavitation.

$$W = \frac{\text{Inertia force}}{\text{Surface tension}} = \frac{\rho V^2 L}{\sigma}$$

We remember it in these Euler numbers inertia force is in the below okay as compared to the other component. So we have a 4 numbers with us the Reynolds numbers, Froude numbers, Weber numbers and Euler number these cavitation which happens when your turbine moving it we can see that there is a chance to have the cavitation formation near the turbine okay. So but there are the conditions where the both the things are dominance that dominates like a surface tension and the cavitation which is happens it is very interesting stuff.

$$Eu = \frac{\text{Pressure force}}{\text{Inertia force}} = \frac{-\Delta p}{\rho V^2}$$

Like that if a bullet traveling in waters it surface tensions as well as the pressure drops creates the cavitation process. So we can see this the bubble formations and all these formations are also collapsing of the bubbles as the bullet travels it which because the inertia force is the dominance is there. Surface tension dominance is there also there is a dominance of the pressure force drops which generates the cavitation process.

That is the way, there are the water vapour formation are there and that workforce are collapsing it is that process if you look at the very interesting process what is happening it just firing a bullet in water that means to the process are happening in say dominance with respect to the inertia forces we have to do dominance f that.

**(Refer Slide Time: 35:19)**

Dimensionless groups in fluid dynamics

- Mach number
 
$$M = \frac{\text{Flow speed}}{\text{Sound speed}} = \frac{V}{a}$$



Boeing flight



Domestic Flight  
Boeing 737-Max  
Mach number: 0.785



Fighter jet: SR-71 black bird  
Mach number: 3.35
- Drag coefficient
 
$$C_D = \frac{\text{Drag force}}{\text{Dynamic force}} = \frac{D}{\frac{1}{2} \rho V^2 A}$$





Similarly, Prandtl number, Rossby number, Specific-heat ratio, Roughness ratio, Rayleigh number, Pressure Coef etc.,

$$M = \frac{\text{Flow speed}}{\text{Sound speed}} = \frac{V}{a}$$

Now you can look in there another 2 components which is a Mach number flow speed by the sound speed okay this numbers becomes 1 when the flow speed is equal to the sound of the speed okay? If you look at the Mach number of Boeing 737 flight max will be 0.785 which is the subsonic flow but if you talk about the fighter jet Sr 1 black bird the Mach number is 3.35 that means the pure velocity its much larger than the speed of this jet fighter jet is larger than the speed of the sound.

You can understand it but where but in this is not the case in case of your domestic flight but the fighter flight knows that. So you need a different technology and all the things if you are really interested just google it and get it what the appropriate for the fighter jet and what is appropriate for the Boeing 737. So these are the 2 different Mach numbers and the low pattern changes it the compressibility changes it in both the cases the compression matters because when Mach number more than 0.3 we talk about the compressibility.

So both the cases we have the low compressibility so that way we can know it that with respect to Mach numbers the flow speed and the sound speeds okay with aerodynamics and more advanced level you will know it. So you can move it but is the range of style and in other words you know it as we discuss it the drag coefficient which is,

$$C_D = \frac{\text{Drag force}}{\text{Dynamic force}} = \frac{D}{\frac{1}{2}\rho V^2 A}$$

and just substitute the dimensions and you can find out is a force component when you have a  $\rho V^2 A$ .

Why does the half is there please you think it so if you have a drag force we say earlier that today technology has developed and all we do the modelling at the full scale levels the prototype levels if you at these things and with a smoke or a jet exactly streamlined patterns how it happened at different velocity and the drag force component? So this is what its designed for speed to moving more than 200 kilometre per hours.

That is the reason even if there is a cyclone we cannot have a rolling but what happened to the bus? It has not designed for the 200 kilometre per hours. So that is the reason so you can see them rolling of the bus in the cyclone situations but that will not happen in case of well-designed car what do you have because these are all designed for high speeds. The design velocity itself is close to 200 kilometre per hour.

So we will not have a rolling conditions what we had seen it bus is ruling conditions when you have it the cyclones that is what we mean out there because it is a well-tested at the prototype levels it is not in the model level it is a prototype levels and to find out what is the drag force mesh under drag force it has designed all the components with the width should be there where the lightweight will be there.

Whether how many passengers are sitting inside or not sitting inside all the condition they tested it and to find out the best the safe which even an extreme design conditions of the speed of relatively 200 kilometre per hours. Still this will be the not going to be ruling conditions what we saw the ruling of a bus in cyclone Fani.

**(Refer Slide Time: 39:10)**



Buckingham's  $\pi$ -theorem

Example 2

Develop the independent dimensionless group using Buckingham's  $\pi$ -theorem

from  $Q = f\left(R, \mu, \frac{dp}{dx}\right)$

Independent dimensionless groups:

Variables:	$R, \mu, \frac{dp}{dx} \text{ and } Q \ (n = 4)$	} = $n - j = 4 - 3 = 1$ number independent dimensionless groups from pi theorem
Basic Dimensions:	$M, L \text{ and } T \ (r = 3)$	
Repeating Variables:	$R, \mu \text{ and } \frac{dp}{dx} \ (j = 3)$	

Dimensions of each variable:

$Q$	$R$	$\mu$	$\frac{dp}{dx}$
$L^3 T^{-1}$	$L$	$M L^{-1} T^{-1}$	$M L^{-2} T^{-2}$

So for let us coming to the closure to this before that let us have it 2 or 3 examples which are very easy things I will not repeat much things that let us have a find it an independent of dimensional group using there is a Q which is a volumetric discharge is a function of r radius dynamic viscosity and the gradient of the pressure okay.

$$Q = f\left(R, \mu, \frac{dp}{dx}\right)$$

We have n 4, j we know the 3 mass ,length and time because all these things as soon as you write the dimensional discharge R,  $\mu$ , and dp/dx.

Variables:  $R, \mu, \frac{dp}{dx} \text{ and } Q \ (n = 4)$

Basic Dimensions:  $M, L \text{ and } T \ (r = 3)$

Repeating Variables:  $R, \mu \text{ and } \frac{dp}{dx} \ (j = 3)$

Remember this is a dp/dx that means find the pressure, dimension divide by the length dimensions you will get a dp/dx decision. So once you know this time instance as you know that it will form a 1 independent dimensional group.

**(Refer Slide Time: 40:07)**

Example 2

$$\pi_1 = R^a \mu^b \frac{dp}{dx}^c Q$$

$$(L)^a (M^1 L^{-1} T^{-1})^b (M^1 L^{-2} T^{-2})^c (L^3 T^{-1}) = M^0 L^0 T^0$$

Solving,  $a=-4, b=1, c=-1$

$$\pi_1 = R^{-4} \mu^1 \frac{dp}{dx}^{-1} Q = \frac{Q \mu^1}{R^{-4} \frac{dp}{dx}} = \text{Constant}$$

$$\frac{Q \mu^1}{R^{-4} \frac{dp}{dx}} = \text{Constant}$$

We will follow the same concept I am not going to repeat it here again you put a substitutes of power x point of abc substitute this dimensional values and equate with non-dimensional value you will get abc value and finally we will get a the relationship between the 4 which will make a non-dimensional group it is a very example simple things but first thing is that we should remember the dimensions of the fluid variables.

$$\pi_1 = R^a \mu^b \frac{dp}{dx}^c Q$$

$$(L)^a (M^1 L^{-1} T^{-1})^b (M^1 L^{-2} T^{-2})^c (L^3 T^{-1}) = M^0 L^0 T^0$$

Solving,  $a=-4, b=1, c=-1$

$$\pi_1 = R^{-4} \mu^1 \frac{dp}{dx}^{-1} Q = \frac{Q \mu^1}{R^{-4} \frac{dp}{dx}} = \text{Constant}$$

$$\frac{Q \mu^1}{R^{-4} \frac{dp}{dx}} = \text{Constant}$$

(Refer Slide Time: 40:40)

Buckingham's  $\pi$ -theorem

Example 3

Develop the independent dimensionless group using Buckingham's  $\pi$ -theorem from  $\delta = f(P, L, I, E)$

Independent dimensionless groups:

Variables:	$P, L, I, E$ and $\delta$ ( $n = 5$ )	} = $n - j = 5 - 2 = 3$ number independent dimensionless groups from pi theorem
Basic Dimensions:	$M, L$ and $T$ ( $r = 3$ )	
Repeating Variables:	$L$ and $E$ ( $j = 2$ )	

Dimensions of each variable:

$\delta$	$P$	$L$	$I$	$E$
$L^1$	$M^1 L^1 T^{-2}$	$L^1$	$L^1$	$M^1 L^{-1} T^{-2}$

And this is another example 3 is where what the basic differences here is that we have a dimensional groups we have to make it with a conditions this is not Froude problems we have a variable which is E value is there and you have a P you have length you have a deflection something like you have a deflection. So you have a length dimensions you have pressure the dimensions you have length I and you have an even.

In this case what you have to look at that when you have this type of 2 conditions where you have a having this mass so we cannot make a 3 variable combine it to make a non-dimensional only the 2 variable is considered to make a non-dimensional problem. So our instead of 3 we have used 2 because 3 variable we cannot combine it to make a non-dimensional problem. So because only these 2 variables here having a dimensions where the mass is there. So in that case you will consider  $j =$  or  $r=2$  so you make a 3 independent dimensional groups from a pi theorem.

Variables:  $P, L, I, E$  and  $\delta$  ( $n = 5$ )

Basic Dimensions:  $M, L$  and  $T$  ( $r = 3$ )

Repeating Variables:  $L$  and  $E$  ( $j = 2$ )

**(Refer Slide Time: 42:03)**

Example 3

$\pi_1 = L^a E^b I$	$\pi_2 = L^a E^b P$	$\pi_3 = L^a E^b \delta$
$(L)^a (M^1 L^{-1} T^{-2})^b (L^4) = M^0 L^0 T^0$	$(L)^a (M^1 L^{-1} T^{-2})^b (M^1 L^1 T^{-2}) = M^0 L^0 T^0$	$(L)^a (M^1 L^{-1} T^{-2})^b (L^1) = M^0 L^0 T^0$
Solving, $a=-4, b=0$	Solving, $a=-2, b=-1$	Solving, $a=-1, b=0$
$\pi_1 = L^{-4} E^0 I^1 = \frac{I}{L^4}$	$\pi_2 = L^{-2} E^{-1} P^1 \mu = \frac{P}{E^1 L^2}$	$\pi_3 = L^{-1} E^0 \delta^1 = \frac{\delta}{L}$
$\frac{\delta}{L} = f\left(\frac{P}{EL^2}, \frac{I}{L^4}\right)$		

That is what the difference now you can look at these tables we look at the  $\pi_1 \pi_2 \pi_3$  we take 2 repeating variables okay and 3 non-repeating variable substitutes all these panels solving it getting the values  $E^b$  finally we get a relationship is that. So if you are good in a solid mechanics you can interpret it what they are okay it is so very easy to interpret it this part because you can know it what they are okay it is not very difficult.

$$\pi_1 = L^a E^b I$$

$$(L)^a (M^1 L^{-1} T^{-2})^b (L^4) = M^0 L^0 T^0$$

Solving,  $a=-4, b=0$

$$\pi_1 = L^{-4} E^0 I^1 = \frac{I}{L^4}$$

$$\pi_2 = L^a E^b P$$

$$(L)^a (M^1 L^{-1} T^{-2})^b (M^1 L^1 T^{-2}) = M^0 L^0 T^0$$

Solving,  $a=-2, b=-1$

$$\pi_2 = L^{-2} E^{-1} P^1 \mu = \frac{P}{E^1 L^2}$$

$$\pi_3 = L^a E^b \delta$$

$$(L)^a (M^1 L^{-1} T^{-2})^b (L^1) = M^0 L^0 T^0$$

Solving,  $a=-1, b=0$

$$\pi_3 = L^{-1} E^0 \delta^1 = \frac{\delta}{L}$$





$$\frac{\delta}{L} = f\left(\frac{P}{EL^2}, \frac{I}{L^4}\right)$$

So this is moment of inertia okay that you can interpret it what these are the deflections values and all so I am trying to do is that the dimensional analysis would not do for fluid mechanics any experiment we do these dimensional analysis to know the relationship between dependant variables and the independent variable in terms of dimensionless formats.



**(Refer Slide Time: 43:05)**

# Wright Brothers Flight


- The Wright Brothers developed world's first successful airplanes like **Wright Flyer** and **Wright Flyer III**.




- The Wright Brothers closely followed bird's wings for balance and control and successfully developed "wing warping".



- Wilbur flew their first plane for 59 seconds at a height of 852 feet in 1903.



**Wilbur Wright**  
1867–1912



**Orville Wright**  
1871–1948

Sources:

[https://www.nasa.gov/images/content/6255main\\_1903flyer.jpg](#)

[https://www.kidzania.co.uk/Wright-Brothers](#)

[https://en.wikipedia.org/wiki/Wright\\_flyer](#)

[http://www.ignite.com.au/ignite/ignite-projects/ignite-project-1-the-wright-flyer-1903.html](#)

Wing Warping design by Wright Brothers

So with this let us have a the closing this lectures before this I think today what do we have advantage to fly from one place to other place by aircraft? It is possible because of these 2 really adventurous people in the world in early 19th century they tried to do it something just looking the nature they had an if you are really interested to know this you just put it the Wright brothers google it and you can know it what way they struggled their life how many failures they faced it.

Before get a succeed to have it what they have succeeded their first plane they succeeded with a 59 seconds in the air at the height of 852 feet okay and because of their success okay now we are having the aircrafts and all because people the human being felt it was easy for now to fly it but what they did it how did they success?

These are not that well educate on this but they are adventurous spirit a spirit they wish to look they had a series of the failures they left their country came to other country and in that period I

will tell it that the Europe universities was were looking to develop something like this top professors are in Germany, Europe all these countries people were trying to look it. But nobody has this type of adventurous what they have that.

So what do you like to tell to these when you do the experiment you do accept the failures? But all the failures sometimes make a success to it. That is what these people proved it and all these failures will give us a opportunity to think why do we have the failures. One thing what they did it just they follow the birds wing for balance and the control which is necessary to fly. Just looking at this bird.

If you look at that it is called wing warping that is what they have designed here which nobody in the university professor could do that thing and that what is the spirit adventurous spirit makes the world in different forms. The basic knowledge because you try you feel it you learn it and that learning spirit makes the things which is something what nobody has invented So with great salute to these 2 brothers and someone who has a very so with this we conclude this lectures and you know it what is the dimension.

(Refer Slide Time: 45:56)

Summary of the Lecture

1. Dimensions of Fluid Mechanics Properties

- Viscosity
- Pressure, stress
- Surface tension
- Force

$M^1L^{-1}T^{-1}$

$M^1L^{-1}T^{-2}$

$M^1L^0T^{-2}$

$M^1L^1T^{-2}$

2. Dimensional Homogeneity Principle

- All equations in the engineering must be dimensionally homogeneous

3. Buckingham's Pi Theorem and Examples

4. Dimensional Groups in Fluid Mechanics

- Reynold's Number, Froude Number, Weber Number, Euler Number and Mach Number

- Viscosity  $M^1 L^{-1} T^{-1}$
- Pressure, stress  $M^1 L^{-1} T^{-2}$
- Surface tension  $M^1 L^0 T^{-2}$
- Force  $M^1 L^1 T^{-2}$

What is the dimension homogeneity fluid flow and what is the dimensional groups in the fluid mechanics will repeat these things in the next class with tis let us I think you for your attention for this?